Design of Discrete-Time Sliding Mode Controller for Minimum Phase Modified Quadruple Tank Process Control System

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Abstract - This paper presents Design of Discrete-Time Sliding Mode controller for Minimum Phase Modified Quadruple Tank Process Control System. The new general form of plant model is developed for illustrating the highly flexible plant structure which can be adjusted to many styles that are used for the propose of giving control system engineers experience with multivariable control system design. This paper described about structure and physical properties of Modified Quadruple-Tank Process, Mathematical plant modeling, Analysis of plant transfer matrix characteristic and here it is proposed a new controller design based on “Sliding Mode Control Strategies”, which has the inherent characteristic and guarantee for the disturbance rejection and robustness.

Keywords: Process Control System, Modeling, Discrete-Time Sliding Mode control

I. INTRODUCTION

Most of industrial control problems are nonlinear and have multiple controlled variables that are common properties for the models of industrial processes to have significant uncertainties, strong interactions, and non-minimum phase behavior so it is important for control system engineer, chemical engineer to understand the non-idealities of industrial processes by carrying out experiments with a good laboratory apparatus [1]. A Modified quadruple tank process was designed and constructed to give control system engineers laboratory experience with key multivariable control concepts. The general form of plant model creating can keep all the properties of existing quadruple tank about multivariable zero locations and their directions of transfer matrix G which have intuitive physical interpretations in terms of how the valves γ1 and γ2 are set [2].

One of the main problems with mathematical models of physical systems is that the parameters used in the models cannot be determined with absolute accuracy. Inaccurate parameters can arise from many different factors. The values of parameters may change with time or various effects. These differences existing between the actual system and system model is called uncertainty [3][4].

However, the actual system parameters may change during operation or the input signal takes too large. In these cases, the linear model is no longer representing the actual system and causes practical problems. Therefore, a robust controller is needed to stabilize these types of systems for the entire range of expected variations in the plant parameters.

The remainder of this paper is organized as follows: Section 2 discusses the details of modified quadruple tank system. In Section 3, the model development is done. Section 4 represents the transfer function matrix of the system. In Section 5 represents the structure of PI controller design. Section 6 shows simulation results followed by conclusion in Section 7.

II. MODIFIED QUADRUPLE TANK PROCESS

The modified quadruple tank process is a combination of two double tank system is shown in Fig.1 serial ports.

This setup consist of a water supply tank with two variable speed positive displacement pump (capacity 0-200V, 1ph) for water circulation fitted with flow dampers, four transparent process tanks fitted with level transmitters, rotameters. Process signals from the four tank level transmitters are interfaced with computer. Control algorithm running on the computer sends output to the individual pump variable frequency drive through interfacing units. Tank 1 and Tank 2 are mounted below the other two tanks for receiving water flow by gravity. Each tank outlet opening is fitted with a valve. The connected valve between tank 1 and tank 2 combines water flow path of tank 1 and tank 2. Both pumps 1 and 2 suction from the supply tank. Each pump is fitted with an air cushioned buffer tank to dampen the flow fluctuations from the metering pumps. Discharge from pump 1 is split between tank 1 and tank 3 and flows are indicated be rotameters 1 and 3. Similarly, pump 2 splits its discharge between tank 2 and tank 4 and the split flows are indicated by rotameters 2 and 4. Split of flow from pump 1 and pump 2 can be varied by manual adjustment of valves S1 and S2. Tank 1 and Tank 2 also receive gravity flow from tank 3 and tank 4, respectively. A connected valve V5 in fig. 1, it combines the water flow path of tank1 with tank2. Opening of these valves (V1, V2, V3, V4 and V5), and the flow split valves (S1 and S2).
can be manually adjusted to substantially alter the characteristics of the system. When the connected valve ratio $\beta$ is taken to be 0.1, it will create the interacting channel between water process in tank 1 and tank 2. By the interacting structure, we can assess the performance of control system design in the interacting condition.

What makes the process more complicated is the dependence of split valve opening. The split fraction of flow from a pump going to the lower tank decreases with increase in pump flow. This is a strong source of non-linearity and the process initially starting in minimum phase can transit to non-minimum phase during operation. The Quadruple tank process has two transmission zeros. The position of one of these zeros depends on split fraction $\gamma_1$ and $\gamma_2$ in valves 1 and 2 respectively.

The minimum and non-minimum phase mode can be achieved as

Minimum Phase: $1 < (\gamma_1 + \gamma_2) < 2$
Non-minimum Phase: $0 < (\gamma_1 + \gamma_2) < 1$

III. MODEL DEVELOPMENT

The control objective of modified quadruple tank system is to control water level in two lower tanks i.e. tank 1 and tank 2.

The process has two inputs – flow from pump 1 and pump 2. These are set by signal inputs $u_1$ and $u_2$, which are the output from the controller. There are four levels ($h_1, h_2, h_3$ and $h_4$) that are measured, transmitted and are available on-line for the control algorithm to make use of it.

Mass balance and Bernoulli’s law yield non-linear plant equation as following [5][6].

\[
\frac{dh_1(t)}{dt} = \frac{\gamma_1 k_{p1}}{A} u_1(t) + \frac{\beta_{a2}}{A} \sqrt{2gh_2(t)} - \frac{\beta_{a1}}{A} \text{sgn}(h_1(t)-h_2(t)) \sqrt{2gh_1(t) - h_2(t)}
\]

\[
\frac{dh_2(t)}{dt} = \frac{\gamma_1 k_{p2}}{A} u_2(t) + \frac{\beta_{a2}}{A} \sqrt{2gh_4(t)} + \frac{\beta_{a2}}{A} \text{sgn}(h_1(t)-h_2(t)) \sqrt{2gh_1(t) - h_2(t)} - \frac{\beta_{a1}}{A} \sqrt{2gh_2(t)}
\]

\[
\frac{dh_3(t)}{dt} = \frac{(1-\gamma_1) k_{p1}}{A} u_1(t) - \frac{\beta_{a2}}{A} \sqrt{2gh_3(t)}
\]

\[
\frac{dh_4(t)}{dt} = \frac{(1-\gamma_1) k_{p2}}{A} u_2(t) - \frac{\beta_{a2}}{A} \sqrt{2gh_4(t)}
\]

Where

$A$ : Cross section area of tank = 30 cm$^2$

$a_1$ : Cross section area of the outlet hole (cm$^2$)

$h_1(t)$ : water level (cm)

$u_1(t)$ : Voltage input of pump (volt)

$\beta_i$ : Outlet valve ratio

$\gamma_1$ : connected valve ratio between tank 1 and tank 2

$\gamma_2$ : Inlet valve ratio

For minimum phase: $\gamma_1 = 0.70$ & $\gamma_2 = 0.60$

For Non-minimum phase: $\gamma_1 = \gamma_2 = 0.3$

$k_{pi}$ : Gain of pump (cm$^2$/volt/sec)

$g$ : Specific gravity = 981 cm/sec$^2$

The pump generate a flow proportional to the applied voltage:

$q_{pump,i}(t) = k_{pi} \cdot u_i(t), \text{[where j=1.2 & i=1.2]}$

The flow that split up by the valves to tank 1 is $\gamma_1 k_{p1} u_1(t)$, tank 2 is dynamics. $\gamma_2 k_{p2} u_2(t)$, tank 3 is $(1-\gamma_1) k_{p1} u_1(t)$, tank 4 is $(1-\gamma_2) k_{p2} u_2(t)$. Each of the valves (V1, V2, V3, V4, V5, S1 and S2) has non-linear characteristics and they interact to increase the order of dynamics.

IV. DESIGN OF A DISCRETE-TIME SLIDING MODE CONTROLLER

a) Discrete-time Sliding Mode Control

A discrete-time version of sliding mode control is important when the implementation of the control is realized by computers with a relatively slow sampling period [7].

Consider the following single input discrete-time system:

$x(k+1) = Ax(k) + bu(k)$

in which, $x(k)$ is the state vector, $u(k)$ is the control input and $A$, $b$ are system and input matrices of appropriate dimensions, respectively. When a DSMC is applied to this system, the state response of the system can be separated into the Reaching Mode (RM), Sliding Mode (SM), and Steady-State (SS) modes as shown in Fig. 2. Fig. 2 (a) shows trajectories of a continuous-time variable structure control (VSC) system, and Fig. 2 (b) shows trajectories of a discrete-time VSC system. Type-I trajectory is an ideal trajectory, whereas type-P trajectory represents the state motion for a practical discrete-time VSC system [7][8]. The state trajectory of a discrete-time VSC system moves monotonically towards the switching plane and cross it in finite time. Once the trajectory crosses the switching plane the first time, it crosses the plane again in every successive sampling period, resulting in a zigzag motion about the switching plane. The size of each successive zigzagging step is non-increasing and the trajectory stays within a specified band called quasi-sliding mode (QSM) band [7].

![Fig. 2(a) Trajectories of a continuous-time VSV system. (b) Trajectories of a discrete time VSC system](image)

b) Linearization of the Modified Quadruple Tank System

We represent the system by a state space form:

\[\dot{X} = AX + Bu\]

\[y = CX + Du\]

The System has $h = f(h,u)$, Where $f$ is non-linear function of water level $h$ and the pump voltage $u$ so we have to linearize the system around the steady state $(h, u)$ for the state
representation. From the aid of Taylor series expansion, Linearizing the non-linear system has represented by a state space system as following.

\[
\begin{bmatrix}
X_1 \\
X_2 \\
X_3 \\
X_4
\end{bmatrix} = 
\begin{bmatrix}
\frac{-1}{\tau_1} & \frac{1}{\tau_x} & 0 & 0 \\
\frac{1}{\tau_x} & \frac{-1}{\tau_1} & \frac{1}{\tau_2} & 0 \\
0 & 0 & 0 & -\frac{1}{\tau_x} \\
0 & 0 & 0 & -\frac{1}{\tau_x}
\end{bmatrix}
\begin{bmatrix}
X_1 \\
X_2 \\
X_3 \\
X_4
\end{bmatrix} + 
\begin{bmatrix}
\gamma_1 k_{p1} \\
\gamma_2 k_{p2} \\
(1-\gamma_2) k_{p2} \\
\gamma_1 k_{p1}
\end{bmatrix} [u_1] + 
\begin{bmatrix}
\gamma_2 k_{p2} \\
\gamma_1 k_{p1} \\
0 \\
0
\end{bmatrix} [u_2]
\] (6)

Where the time constant:

\[
\frac{1}{\tau_1} = \frac{b_{z1}}{A} \left[ \frac{g}{2h_1} \right] i = 1,\ldots,4
\]

\[
\frac{1}{\tau_x} = \frac{b_{z2x}}{A} \left[ \frac{g}{2[b_1 - b_2]} \right]
\] (7)

In equations (6) \( u \) represents the control input after linearization of the system. This linear model is subsequently discretized with sampling time \( \tau = 0.1 \) sec, which is suitably chosen to get the best system response and minimum chattering. Thus, the continuous-time system as given in (6) is converted to a discrete time system

\[
x(k+1) = Ax(k) + bu(k)
\]

in which,

\[
Ae^{\Delta t} = 
\begin{bmatrix}
0.9941 & 1.0043 & 1.0042 & 1 \\
1.0043 & 0.9946 & 1 & 1.0033 \\
1 & 1 & 0.9958 & 1 \\
1 & 1 & 1 & 0.9967
\end{bmatrix}
\]

\[
b = \lambda^{-1}[A-I]b = 
\begin{bmatrix}
-11.6106 \\
-13.3414 \\
-1.8533 \\
-3.3234
\end{bmatrix}
\] (8)

c) Switching Surface Design

After linearization and discretization of the system, the system states are made to confine to a certain subspace of the state space termed as the switching surface which is designed as follows:

\[
s(k) = c^T x(k) = [c_1 \ c_2 \ c_3 \ c_4] \begin{bmatrix}
X_1 \\
X_2 \\
X_3 \\
X_4
\end{bmatrix}
\] (9)

Coefficients, \( c_1, c_2, c_3, c_4 \) are arbitrarily chosen such that the closed loop poles are located at in the left half of the s-plane and the system is stable.

d) Formulation of the control law

Next step in the design of the DSMC is formulation of the controller using Gao’s discrete time reaching law as given below [2]:

\[
s(k+1) - s(k) = -q s(k) - \varepsilon \text{sgn}(s(k))
\] (10)

\( \varepsilon > 0, q > 0, 1-q\tau > 0 \)

in which, \( \varepsilon, q \) are non zero constants, and \( \tau \) is the sampling time. On substituting \( s(k) \), and hence \( s(k+1) \) from (9), and \( x(k+1) \) from (8) in eqn. (10) and solving for \( u(k) \), we get the discrete time sliding mode control law as follows:

\[
u(k) = -(c^T b)^{-1} \{ c^T Ax(k) - c^T \dot{x}(k) + q \varepsilon \text{sgn}(c^T x(k)) \}
\] (11)

The response of the discrete VSC system

\[
x(k+1) = Ax(k) - b(c^T Ax(k) - c^T \dot{x}(k) + q \varepsilon \text{sgn}(c^T x(k))
\]

\[
+ \varepsilon \text{sgn}(c^T x(k))
\] (12)

V. SIMULATION RESULTS

The high frequency oscillation of the control input about its desired reference value is called chattering. Since switching control at an infinite rate is impossible, high frequency chattering always occurs in the sliding and steady state modes of a practical, continuous-time VSC system [1, 2]. Chattering is significantly reduced in DSMC because of the QSM of the state trajectory which serves as a boundary layer.

VI. CONCLUSION

This paper proposes the design of a DSMC for a Modified Quadruple Tank Process Control System. The linearization result, we conclude that the Modified quadruple Tank system is stable. And Design of DSMC gives the robustness of the system.

VII. REFERENCES


